

# Novelty Assessment Report

**Paper:** Multilevel Control Functional

**PDF URL:** <https://openreview.net/pdf?id=Ahdsq2nkNH>

**Venue:** ICLR 2026 Conference Submission

**Year:** 2026

**Report Generated:** 2025-12-27

## Abstract

Control variates are variance reduction techniques for Monte Carlo estimators. They play a critical role in improving Monte Carlo estimators in scientific and machine learning applications that involve computationally expensive integrals. We introduce \emph{multilevel control functionals} (MLCFs), a novel and widely applicable extension of control variates that combines non-parametric Stein-based control variates with multi-fidelity methods. We show that when the integrand and the density are smooth, and when the dimensionality is not very high, MLCFs enjoy a faster convergence rate. We provide both theoretical analysis and empirical assessments on differential equation examples, including Bayesian inference for ecological models, to demonstrate the effectiveness of our proposed approach. Furthermore, we extend MLCFs for variational inference, and demonstrate improved performance empirically through Bayesian neural network examples.

### Disclaimer

This report is **AI-GENERATED** using Large Language Models and WisPaper (a scholar search engine). It analyzes academic papers' tasks and contributions against retrieved prior work. While this system identifies **POTENTIAL** overlaps and novel directions, **ITS COVERAGE IS NOT EXHAUSTIVE AND JUDGMENTS ARE APPROXIMATE**. These results are intended to assist human reviewers and **SHOULD NOT** be relied upon as a definitive verdict on novelty.

Note that some papers exist in multiple, slightly different versions (e.g., with different titles or URLs). The system may retrieve several versions of the same underlying work. The current automated pipeline does not reliably align or distinguish these cases, so human reviewers will need to disambiguate them manually.

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## Core Task Landscape

This paper addresses: **Variance Reduction for Monte Carlo Integration Using Multilevel Control Functionals**

A total of **21 papers** were analyzed and organized into a taxonomy with **9 categories**.

### Taxonomy Overview

The research landscape has been organized into the following main categories:

- **Multilevel Monte Carlo Methods with Control Variates**
- **Multilevel Monte Carlo Frameworks**
- **Control Variate Techniques**
- **Domain-Specific Applications**
- **Survey and Overview Literature**

### Complete Taxonomy Tree

- Variance Reduction for Monte Carlo Integration Using Multilevel Control Functionals Survey Taxonomy
- Multilevel Monte Carlo Methods with Control Variates
  - Multifidelity and Surrogate-Based Control Variates (3 papers)
    - [4] A multifidelity control variate approach for the multilevel Monte Carlo technique (G Geraci, 2015) [View paper](#)
    - [6] Multilevel surrogate-based control variates (Amri, 2023) [View paper](#)
    - [7] Covariance Expressions for Multifidelity Sampling with Multioutput, Multistatistic Estimators: Application to Approximate Control Variates (Thomas O. Dixon, 2023) [View paper](#)
  - Multilevel Control Variates for Partial Differential Equations (4 papers)
    - [13] A Multi Level Monte Carlo method with control variate for elliptic PDEs with log-normal coefficients (Fabio Nobile, 2015) [View paper](#)
    - [16] Uncertainty quantification for the BGK model of the Boltzmann equation using multilevel variance reduced Monte Carlo methods (Jingwei Hu, 2020) [View paper](#)
    - [18] An empirical interpolation and model-variance reduction method for computing statistical outputs of parametrized stochastic partial differential equations (Michael B. Giles, 2016) [View paper](#)
    - [19] A multilevel Monte Carlo reduced basis method for the hdg approximation of stochastic elliptic partial differential equations (F Vidal-Codina, n.d.) [View paper](#)
  - General Multilevel Control Variate Methods ★ (4 papers)
    - [0] Multilevel Control Functional (Anon et al., 2026) [View paper](#)
    - [14] A Low-rank Control Variate for Multilevel Monte Carlo Simulation of High-dimensional Uncertain Systems (Fairbanks, 2022) [View paper](#)
    - [17] A weighted multilevel Monte Carlo method (Yu, 2024) [View paper](#)
    - [20] A multilevel approach to control variates (Speight, 2009) [View paper](#)
- Multilevel Monte Carlo Frameworks
  - Adaptive Multilevel Monte Carlo (2 papers)
    - [11] Implementation and analysis of an adaptive multilevel Monte Carlo algorithm (HÅkon Hoel, 2012) [View paper](#)
    - [15] Adaptive multilevel monte carlo simulation (HÅkon Hoel, 2011) [View paper](#)
  - Multilevel Monte Carlo for Metamodelling and Bayesian Quadrature (2 papers)
    - [3] Multilevel Monte Carlo Metamodelling for Variance Function Estimation (Zhang Jingtao, 2025) [View paper](#)
    - [9] Multilevel Methods for Monte Carlo Integration, with Applications to Tsunami Modelling (KaiYu, 2024) [View paper](#)
- Control Variate Techniques (1 papers)
  - [5] Multiscale variance reduction methods based on multiple control variates for kinetic equations with uncertainties (Giacomo Dimarco, 2020) [View paper](#)

- Domain-Specific Applications
  - Financial Risk and Option Pricing (2 papers)
  - [8] Sub-sampling and other considerations for efficient risk estimation in large portfolios (Giles, 2019) [View paper](#)
  - [10] On Multilevel and Control Variate Monte Carlo Methods for Option Pricing under the Rough Heston Model (Siow Woon Jeng, 2021) [View paper](#)
  - Computational Physics and Engineering (3 papers)
  - [2] Variance Reduction in Trace Estimation for Lattice QCD Using Multigrid Multilevel Monte Carlo (Jose Jimenez-Merchan, 2025) [View paper](#)
  - [12] Multilevel control variates for uncertainty quantification in simulations of cloud cavitation (Jonas Åukys, 2018) [View paper](#)
  - [21] Metropolis Methods for Quantum Monte Carlo Simulations (Ceperley, 2003) [View paper](#)
- Survey and Overview Literature (1 papers)
  - [1] Modern Monte Carlo methods for efficient uncertainty quantification and propagation: A survey (Zhang, 2021) [View paper](#)

## Narrative

Core task: variance reduction for Monte Carlo integration using multilevel control functionals. The field addresses the challenge of efficiently estimating expectations when direct Monte Carlo sampling is prohibitively expensive or high-variance. The taxonomy reveals several complementary directions. One major branch, Multilevel Monte Carlo Methods with Control Variates, focuses on combining hierarchical sampling across multiple discretization levels with auxiliary functions (control variates) to cancel variance. A second branch, Multilevel Monte Carlo Frameworks, emphasizes the algorithmic scaffolding—adaptive level selection, convergence diagnostics, and computational cost balancing—that underpins practical implementations such as Adaptive MLMC Algorithm[11] and Adaptive Multilevel Monte Carlo[15]. A third branch, Control Variate Techniques, explores the construction and optimization of these auxiliary functions in isolation, including low-rank approximations and surrogate modeling strategies. Domain-Specific Applications then demonstrate how these methods translate to concrete settings like financial option pricing (MLMC Rough Heston[10]), natural hazards (Multilevel Monte Carlo Tsunami[9]), and computational physics (Variance Reduction Lattice QCD[2]). Finally, Survey and Overview Literature, exemplified by Modern Monte Carlo Survey[1], synthesizes methodological advances and contextualizes emerging trends.

Within the control-variate-enhanced multilevel methods, a particularly active line of work investigates how to construct or learn optimal control functionals at each level. Some studies, such as Multifidelity Control Variate MLMC[4] and Multilevel Control Variates Approach[20], emphasize analytic or model-based surrogates, while others like Lowrank Control Variate MLMC[14] exploit low-rank structure to reduce computational overhead. The original paper, Multilevel Control Functional[0], sits squarely in this cluster, proposing a systematic framework for integrating control functionals across levels. Compared to Weighted Multilevel Monte Carlo[17], which adjusts level weights to balance bias and variance, Multilevel Control Functional[0] instead focuses on constructing auxiliary functions that directly target residual variance at each stage. This emphasis on functional design distinguishes it from purely algorithmic tuning approaches and aligns it closely with works like Multilevel Surrogate Control Variates[6], which similarly leverage surrogate models to enhance variance reduction.

## Related Works in Same Category

The following **3 sibling papers** share the same taxonomy leaf node with the original paper:

### 1. A Low-rank Control Variate for Multilevel Monte Carlo Simulation of High-dimensional Uncertain Systems

**Authors:** Fairbanks, Hillary, Doostan, Alireza, Iaccarino, et al. (6 authors total) | **Year/Venue:** 2022 • arXiv (Cornell University) | **URL:** [View paper](#)

#### Abstract

Multilevel Monte Carlo (MLMC) is a recently proposed variation of Monte Carlo (MC) simulation that achieves variance reduction by simulating the governing equations on a series of spatial (or temporal) grids with increasing resolution. Instead of directly employing the fine grid solutions, MLMC estimates the expectation of the quantity of interest from the coarsest grid solutions as well as differences between each two consecutive grid solutions. When the differences corresponding to finer grids...

#### Relationship Analysis

Both papers belong to the General Multilevel Control Variate Methods category, combining multilevel Monte Carlo with control variates to reduce variance. The candidate paper (Fairbanks et al., 2017) uses low-rank approximations of solutions on coarser grids as control variates for MLMC, while the original paper employs non-parametric Stein-based control functionals (kernel-based methods) that leverage score information and RKHS theory. The key difference is that the original paper's approach is more broadly applicable (works with unnormalized densities, extends to variational inference) and does not require low-rank structure, whereas the candidate paper specifically targets problems where solutions admit low-rank approximations.

### 2. A weighted multilevel Monte Carlo method

**Authors:** Li Yu | **Year/Venue:** 2024 | **URL:** [View paper](#)

#### Abstract

The Multilevel Monte Carlo (MLMC) method has been applied successfully in a wide range of settings since its first introduction by Giles (2008). When using only two levels, the method can be viewed as a kind of control-variate approach to reduce variance, as earlier proposed by Kebaier (2005). We introduce a generalization of the MLMC formulation by extending this control variate approach to any number of levels and deriving a recursive formula for computing the weights associated with the contr...

#### Relationship Analysis

Both papers belong to the General Multilevel Control Variate Methods category, focusing on foundational multilevel approaches for variance reduction in Monte Carlo integration. The candidate paper introduces a weighted MLMC formulation that generalizes the control variate approach across multiple levels through recursive weight computation, overlapping with the original paper's goal of combining control variates with multilevel structures. However, the original paper specifically develops multilevel control functionals using non-parametric Stein-based control variates in RKHS, while the candidate paper focuses on weighted formulations for standard MLMC without the Stein operator framework or kernel-based approaches.

### 3. A multilevel approach to control variates

**Authors:** Adam Speight | **Year/Venue:** 2009 | **URL:** [View paper](#)

#### Abstract

We present a new variance reduction technique that naturally applies to the pricing of financial derivatives by Monte Carlo simulation. Inspired by multigrid methods for solving partial differential equations, the technique is based on control variates derived from a sequence of approximations that converge pathwise to a limiting model. It applies to a large class of problems, and is easy to implement. Theory and computational results show this method can substantially reduce computational time ...

## Relationship Analysis

Both papers belong to the General Multilevel Control Variate Methods category, sharing the foundational approach of combining multilevel structures with control variates for variance reduction in Monte Carlo integration. The candidate paper presents a multilevel control variate framework inspired by multigrid methods for financial derivatives pricing, while the original paper (Multilevel Control Functional) specifically develops a Stein-based non-parametric approach using reproducing kernel Hilbert spaces (RKHS) with theoretical convergence guarantees and extends to variational inference applications. The key difference is that the original paper employs sophisticated kernel-based control functionals with theoretical analysis of optimal sample allocation, whereas the candidate focuses on pathwise convergent approximation sequences for financial applications.

## Contributions Analysis

**Overall novelty summary.** The paper introduces multilevel control functionals (MLCFs), combining non-parametric Stein-based control variates with multi-fidelity methods for variance reduction in Monte Carlo integration. It resides in the 'General Multilevel Control Variate Methods' leaf, which contains four papers including the original work. This leaf sits within the broader 'Multilevel Monte Carlo Methods with Control Variates' branch, indicating a moderately populated research direction. The taxonomy shows nine leaf nodes across 21 papers total, suggesting the field is structured but not overcrowded, with room for methodological innovation in foundational multilevel control variate frameworks.

The taxonomy reveals three sibling leaves within the parent branch: 'Multifidelity and Surrogate-Based Control Variates' (three papers on surrogate models), 'Multilevel Control Variates for Partial Differential Equations' (four papers on PDE-specific applications), and the original paper's leaf. Neighboring branches include 'Multilevel Monte Carlo Frameworks' (adaptive strategies, metamodelling) and 'Control Variate Techniques' (non-multilevel methods). The paper bridges Stein-based control variates—typically studied in isolation—with multilevel hierarchies, connecting the 'Control Variate Techniques' branch to the multilevel paradigm. This positioning suggests the work synthesizes ideas from adjacent but previously separate research streams.

Among 21 candidates examined, the contribution-level analysis shows mixed novelty signals. The core MLCF framework (Contribution 1) examined one candidate with no clear refutation, suggesting limited direct overlap in the search scope. The theoretical variance bounds and sample allocation (Contribution 2) examined ten candidates, with one appearing to provide overlapping prior work, indicating some existing theory in this area. The variational inference extension (Contribution 3) examined ten candidates with no refutations, suggesting this application direction may be less explored. These statistics reflect a focused but not exhaustive literature search, leaving open the possibility of additional relevant work beyond the top-21 semantic matches.

Based on the limited search scope, the work appears to occupy a moderately novel position, particularly in combining Stein-based control variates with multilevel structures and extending to variational inference. The theoretical contribution shows some overlap with existing variance analysis frameworks, while the core method and application extension appear less directly anticipated. The taxonomy structure and sibling papers suggest the field has established foundations but remains open to new integrations, though a broader literature search would be needed to confirm the full extent of novelty across all contributions.

This paper presents **3 main contributions**, each analyzed against relevant prior work:

### Contribution 1: Multilevel Control Functionals (MLCFs)

**Description:** The authors propose MLCFs, a new variance reduction method that integrates non-parametric Stein-based control functionals with multilevel/multi-fidelity structures. This method extends control variates to leverage hierarchical approximations of integrands, achieving faster convergence rates under smoothness assumptions.

This contribution was assessed against **1 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

#### 1. Vector-valued control variates

**URL:** [View paper](#)

##### Brief Assessment

Vector-valued Control Variates[42] focuses on multi-task integration problems using vector-valued functions to share information across related integrals, not on combining control variates with multi-fidelity/multilevel structures for hierarchical approximations.

### Contribution 2: Theoretical variance bounds and optimal sample allocation

**Description:** The authors derive theoretical upper bounds on the variance of MLCF estimators (Theorem 3.2) and establish optimal sample size allocation across fidelity levels (Theorem 3.3) to minimize variance under computational budget constraints.

This contribution was assessed against **10 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

#### 1. Sampling the full hierarchical population posterior distribution in gravitational-wave astronomy

**URL:** [View paper](#)

##### Brief Assessment

Hierarchical Population Posterior Gravitational[32] focuses on hierarchical Bayesian inference for gravitational-wave astronomy using Hamiltonian Monte Carlo, not on deriving variance bounds or optimal sample allocation for multilevel Monte Carlo estimators.

#### 2. An Antithetic Multilevel Monte Carlo-Milstein Scheme for Stochastic Partial Differential Equations

**URL:** [View paper](#)

##### Brief Assessment

Antithetic MLMC Milstein[37] focuses on variance bounds for stochastic partial differential equations using antithetic coupling and Milstein schemes, not on control functional estimators for general Monte Carlo integration as in the original paper.

#### 3. Accelerated multilevel Monte Carlo with kernel-based smoothing and Latinized stratification

**URL:** [View paper](#)

##### Brief Assessment

Accelerated MLMC Kernel Smoothing[39] focuses on CDF estimation for subsurface flow problems using kernel smoothing and stratified sampling, not on deriving general variance bounds or optimal sample allocation theory for MLCF estimators.

#### 4. A multifidelity multilevel Monte Carlo method for uncertainty propagation in aerospace applications

**URL:** [View paper](#)

##### Prior Art Analysis

Multifidelity MLMC Aerospace[34] demonstrates that theoretical variance bounds and optimal sample allocation for multilevel Monte Carlo estimators were established prior to the original paper. The candidate derives variance expressions for multilevel-multifidelity estimators and presents optimal sample allocation formulas that minimize computational cost under variance constraints. These results

include explicit formulas for variance bounds (equations showing variance as a function of samples and model parameters) and optimal sample size allocation across levels (equations 19, 22, 30-31), which directly parallel the theoretical contributions claimed in the original paper.

#### **Evidence**

Evidence 1 - **Rationale**: Both papers derive variance expressions for multilevel estimators, showing how variance decomposes across levels. - **Original**: we provide theoretical variance bounds as well as theoretical optimal sample sizes for mlcf in section 3 - **Candidate**: the variance of  $\hat{q}_{hf,ml}$  is  $\text{var}(\hat{q}_{hf,ml}) = \sum_{\ell=0}^L n_{\ell} \text{var}(y_{hf,\ell})$

Evidence 2 - **Rationale**: The candidate provides an explicit formula for optimal sample allocation across levels, directly addressing the same theoretical problem as the original paper's Theorem 3.3. - **Original**: we provide theoretical variance bounds as well as theoretical optimal sample sizes for mlcf in section 3 - **Candidate**: the optimal samples allocation (per level) as result of the minimization e q. (18) is  $\text{nhf}_{\ell} = 2 \varepsilon^2 [\sum_{k=0}^L (\text{var}(y_{hf,k}) \text{chf}_{\ell,k})^{1/2}] \sqrt{\text{var}(y_{hf,\ell}) \text{chf}_{\ell}}$

Evidence 3 - **Rationale**: Both papers formulate the optimal allocation problem as a constrained optimization using Lagrange multipliers, minimizing cost subject to variance constraints. - **Original**: theorem 3.3. suppose that assumptions a1-a8 hold,  $ml/ml = \rho$  and  $\tau := \tau_L = \min\{al, bl\}$  do not depend on  $l$ . then  $\text{nmlcf}_0, \dots, \text{nmlcf}_L$  is obtained by minimizing  $\sum_{l=0}^L (rlm - \tau l / d_l \|f_l - f_{l-1}\|_{hl} + )^2 / (nl - ml)$  subject to  $\sum_{l=0}^L l \text{chf}_{\ell} = t$  for  $t > 0$  - **Candidate**: the computational cost eq. (17) can be minimized under a variance constrain by means of a lagrange multiplier  $\lambda$ . the minimization problem reads  $\text{argmin}_{\text{nhf}, \lambda} \text{nhf}_{\ell, \lambda}(f)$ , where  $f(\text{nhf}_{\ell, \lambda}) = \sum_{\ell=0}^L \text{nhf}_{\ell, \lambda} \text{chf}_{\ell, \lambda} + \lambda (\sum_{\ell=0}^L (\text{nhf}_{\ell, \lambda}) - 1) \text{var}(y_{hf,\ell}) + \varepsilon^2/2$

Evidence 4 - **Rationale**: Both papers derive theoretical upper bounds on variance for their respective estimators, showing how variance depends on sample sizes and model parameters. - **Original**: theorem 3.2. suppose that the assumptions a1-8 hold and  $x_{1,l}$  are i.i.d at each level, when  $x_{0,l}$  are sufficiently dense, the upper bound of the variance of mlcf estimator is given by  $\text{vx}_{1,0}, \dots, \text{vx}_{1,L} [\text{nmlcf}_l(f)] \leq \sum_{l=0}^L (rlm - \tau l / d_l \|f_l - f_{l-1}\|_{hl} + )^2 / (nl - ml)$  - **Candidate**:  $\text{var}(\hat{q}_{hf, cv, m,n}) = \text{var}(\hat{q}_{hf, m}) (1 - r_1 + r_2 \rho^2)$

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## **5. Milstein schemes and antithetic multilevel Monte Carlo sampling for delay McKeanâ€œVlasov equations and interacting particle systems**

**URL:** [View paper](#)

#### **Brief Assessment**

Milstein Antithetic McKeanVlasov[38] focuses on strong convergence and moment stability for delay McKean-Vlasov equations using Milstein schemes, not on variance bounds or optimal sample allocation for multilevel Monte Carlo estimators.

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## **6. Multi-level monte-carlo gradient methods for stochastic optimization with biased oracles**

**URL:** [View paper](#)

#### **Brief Assessment**

Multilevel MonteCarlo Gradient Biased[33] focuses on stochastic optimization with biased gradient oracles using multi-level Monte Carlo methods, while the original paper addresses variance reduction for Monte Carlo integration using control functionals. The technical settings and problem formulations differ fundamentally.

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## **7. A multilevel Monte Carlo method for performing time-variant reliability analysis**

**URL:** [View paper](#)

#### **Brief Assessment**

MLMC Timevariant Reliability[36] focuses on time-variant reliability analysis with geometric time discretization, not on general variance bounds for multilevel estimators or optimal sample allocation theory across arbitrary fidelity levels.

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## **8. A Separable Bootstrap Variance Estimation Algorithm for Hierarchical Modelâ€œBased Inference of Forest Aboveground Biomass Using Data From NASA's GEDI and Landsat Missions**

**URL:** [View paper](#)

#### **Brief Assessment**

Separable Bootstrap GEDI Landsat[35] focuses on bootstrap variance estimation for forest biomass using GEDI/Landsat data with hierarchical model-based inference, not on deriving theoretical variance bounds or optimal sample allocation for multilevel Monte Carlo estimators.

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## **9. Convergence analysis of multilevel Monte Carlo variance estimators and application for random obstacle problems**

**URL:** [View paper](#)

#### **Brief Assessment**

MLMC Variance Estimators Convergence[41] focuses on variance estimators for random obstacle problems in a multilevel Monte Carlo framework, while the original paper derives variance bounds for multilevel control functionals (combining Stein-based control variates with multi-fidelity methods). These are distinct methodological contributions addressing different aspects of multilevel estimation.

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## **10. Multilevel Monte Carlo Methods for Stochastic Convectionâ€œDiffusion Eigenvalue Problems**

**URL:** [View paper](#)

#### **Brief Assessment**

MLMC Stochastic Eigenvalue[40] focuses on stochastic convection-diffusion eigenvalue problems with finite element discretizations, not general control functional estimators for Monte Carlo integration as in the original paper.

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## **Contribution 3: Extension of MLCFs to variational inference**

**Description:** The authors extend the MLCF framework to variational inference by introducing multilevel control functional re-parameterized gradient (MLCFRG) estimators for optimizing the evidence lower bound (ELBO), with a simplified update form that reduces computational and memory costs.

This contribution was assessed against **10 related papers** from the literature. Papers with potential prior art are analyzed in detail with textual evidence; others receive brief assessments.

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## **1. A Dual Control Variate for accelerated black-box variational inference**

**URL:** [View paper](#)

#### **Brief Assessment**

Dual Control Variate Blackbox[26] focuses on reducing variance in doubly stochastic optimization for black-box variational inference using control variates for both subsampling and Monte Carlo noise. The original paper extends multilevel control functionals (MLCFs) to

variational inference with a multilevel structure across optimization iterations, which is a fundamentally different approach from the dual control variate method that addresses two independent sources of randomness within each iteration.

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## 2. Vargrad: a low-variance gradient estimator for variational inference

URL: [View paper](#)

### Brief Assessment

Vargrad Lowvariance Estimator[28] focuses on gradient estimation for variational inference using control variates based on the score function method, not on multilevel control functionals or multi-fidelity methods for Monte Carlo integration.

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## 3. Pathwise Gradient Variance Reduction with Control Variates in Variational Inference

URL: [View paper](#)

### Brief Assessment

Pathwise Gradient Control Variates[24] focuses on control variates for pathwise gradient estimators in variational inference, while the original paper extends MLCFs (multilevel control functionals) to variational inference with a specific multilevel structure. These are distinct variance reduction approaches for different gradient estimation methods.

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## 4. Variance reduction and quasi-Newton for particle-based variational inference

URL: [View paper](#)

### Brief Assessment

Variance Reduction QuasiNewton Particles[27] focuses on particle-based variational inference methods (like SVGD) using variance reduction and quasi-Newton techniques, not on multilevel control functionals for ELBO optimization.

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## 5. Sticking the landing: Simple, lower-variance gradient estimators for variational inference

URL: [View paper](#)

### Brief Assessment

Sticking Landing Variance[23] focuses on removing the score function term from reparameterized gradients to reduce variance in variational inference, not on extending multilevel control functionals or multi-fidelity methods to VI. The technical approaches are fundamentally different.

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## 6. Extragradient method with variance reduction for stochastic variational inequalities

URL: [View paper](#)

### Brief Assessment

Extragradient Variance Reduction[25] focuses on stochastic variational inequalities using extragradient methods with variance reduction, not on control functionals for variational inference or ELBO optimization.

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## 7. Stochastic variance-reduced Gaussian variational inference on the Bures-Wasserstein manifold

URL: [View paper](#)

### Brief Assessment

Stochastic BuresWasserstein Variational[31] focuses on variance reduction for Gaussian variational inference using control variates in the Bures-Wasserstein manifold geometry, not on multilevel control functionals or multi-fidelity methods for general Monte Carlo integration.

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## 8. GeoPhy: Differentiable Phylogenetic Inference via Geometric Gradients of Tree Topologies

URL: [View paper](#)

### Brief Assessment

GeoPhy Differentiable Phylogenetic[30] focuses on phylogenetic inference using geometric gradients for tree topologies, not on control variates for gradient estimation in general variational inference frameworks.

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## 9. Double control variates for gradient estimation in discrete latent variable models

URL: [View paper](#)

### Brief Assessment

Double Control Variates Gradients[22] focuses on variance reduction for discrete latent variable models using control variates in REINFORCE estimators, not on multilevel control functionals for variational inference. The candidate addresses gradient estimation for discrete variables, while the original extends MLCFs to continuous variational inference settings.

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## 10. Optimal Variance Control of the Score Function Gradient Estimator for Importance Weighted Bounds

URL: [View paper](#)

### Brief Assessment

Optimal Variance Score Function[29] focuses on control variates for score function gradient estimators in importance weighted bounds (IWAE), not on multilevel control functionals for ELBO optimization. The technical approaches differ fundamentally.

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## Appendix: Text Similarity Detection

No high-similarity text segments were detected across any compared papers.

## References

- [0] Multilevel Control Functional [View paper](#)
- [1] Modern Monte Carlo methods for efficient uncertainty quantification and propagation: A survey [View paper](#)
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- [41] Convergence analysis of multilevel Monte Carlo variance estimators and application for random obstacle problems [View paper](#)
- [42] Vector-valued control variates [View paper](#)